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Estimation of Heat-Transfer Coefficient in a Rocket Nozzle

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Nomenclature

a,b,c,d	= coefficients of square temperature matrix
h	= heat-transfer coefficient
k	= thermal conductivity of material
L	= thickness of material
$\dot{q}_c$	= surface heat flux
T	= temperature
T^{n-1}	= temperature at beginning of time step
T^n	= temperature at end of time step
T_g	= combustion gas temperature
T_0	= temperature at surface
t	= time
Δt	= computing time
x	= space coordinate
Δx	= node thickness
α	= thermal diffusivity of material
Subscripts	
$i,0,1$	= node identifier
$0,r$	= surface
j	= thermocouple location
Superscript	
n	= designated the point of time $t+\Delta t$

Introduction

THE determination of the temperature distribution in a rocket nozzle wall subjected to a high-temperature and high-heat-flux environment requires the knowledge of the total heat transferred from the combustion gases. The dominant mode of energy transport in chemical rocket engines is by convection. It is therefore important to estimate accurately the convective heat transfer to the wall in order to achieve an optimum thermal protection system. In heat-transfer studies, many experimental difficulties may arise in implanting heat-flux sensors or thermocouples at the surface for heat-transfer measurements. Furthermore, the presence of a probe at the surface disturbs the condition of the boundary and the flow process adjacent to it and thus actual wall heat flux. In these circumstances it is therefore desirable that the prediction of surface temperature and heat flux be accomplished by inverting the temperature as measured by a probe located interior to the surface of the solid material. Such a problem is termed the inverse problem.

Problems of this kind can be solved using an exact solution,[1] an integral method,[2-5] or a finite-difference method.[6-8] The method sufficiently powerful to solve the general problem appears to be latter, although the basic concepts can be applied to the integral method.

The present Note reports an iterative scheme to obtain values of surface temperature, surface heat flux, heat-transfer coefficient, and combustion gas temperature using a numerical technique in conjunction with the measured temperature history at the outer surface of the rocket nozzle.

Analysis

The physical problem involves a convectively heated slab of finite thickness having a heat sink at one surface and a perfect insulation at the other.

If the temperature on the boundary (surface) node receiving the surface heat can be bounded, then all interior code temperatures will also be included. This boundness must prevent wild oscillation in the surface temperature. This can be achieved by using implicit representation at the surface as

$$\frac{\dot{q}_c}{\Delta x k} - \frac{(T_0^n - T_1^n)}{\Delta x^2} = \frac{1}{2\alpha}\frac{(T_0^n - T_0^{n-1})}{\Delta t} \tag{1}$$

where the subscripts *0* and *1* denote the node identifiers and the superscript n indicates that the value is taken at time $t+\Delta t$.

Since only one-dimensional heat transfer is being considered, the solution can be obtained by solving the tridiagonal system of equations

$$a_i T_{i-1}^n + b_i T_i^n + c_i T_{i+1}^n = d_i \quad \text{for } 0 \le i \le r \tag{2}$$

Rearranging Eq. (1) into this tridiagonal form

$$\left[-2 - \frac{\Delta x^2}{\alpha \Delta t}\right] T_0^n + 2T_1^n + \frac{2\dot{q}_c \Delta x}{k} + \frac{\Delta x^2}{\alpha \Delta t} T_0^{n-1} = 0 \tag{3}$$

The coefficients in Eq. (2) may be readily obtained as

$$a_0 = 0,\ b_0 = -2 - \frac{\Delta x^2}{\alpha \Delta t},\ c_0 = 2, \text{ and } d_0 = -\frac{2\dot{q}_c \Delta x}{k} - \frac{\Delta x^2}{\alpha \Delta t} T_0^{n-1} \tag{4}$$

The tridiagonal system of equations can be solved using the Thomas algorithm.[9] But in the foregoing Eq. (3), $\dot{q}_c$ is an unknown parameter, thus the solution of the complete problem from $x=0$ to x_j cannot be obtained readily because the boundary condition is not known at $x=0$, but rather an interior temperature history is given. In estimating one minimizes

$$F(\dot{q}_c) = [T_c(x_j,t) - T_m(x_j,t)] \tag{5}$$

where T_c and T_m are, respectively, calculated and measured thermocouple temperatures at (x_j,t).

The calculated temperature is, in general, a nonlinear function of $\dot{q}_c$. A simple procedure approximates at each iteration the calculated temperatures by the Newton-Raphson

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Table 1 Comparison of present solution with the Bartz solution

t, s	T_0, K	T_m at outer surface, K	$\dot{q}_c \times 10^6$, W/m^2	h, W/m$^2 \cdot$K	h_B^a,[a] W/m$^2 \cdot$K	T_g, K	T_{gc}, K
6	1355.6	326	5.2502	2631.2	2254.2	3351	2946
7	1287.8	342	3.2950	1805.3	2254.2	3113	2946
8	1315.6	356	3.2974	1861.5	2254.2	3087	2946
9	1368.9	380	3.2967	1885.9	2254.2	3117	2946
10	1414.4	402	3.2837	1962.1	2254.2	3088	2946
11	1463.6	425	3.2718	2049.5	2254.2	3060	2946
12	1370.8	440	2.3825	1476.0	2254.2	2985	2946
13	1360.9	460	2.4140	1502.1	2254.2	2968	2946
14	1370.3	479	2.3625	1520.6	2254.2	2924	2946
15	1382.5	507	2.3675	1517.2	2254.2	2943	2946
16	1399.3	528	2.3645	1540.7	2254.2	2934	2946

[a] h_B^a = heat-transfer coefficient (Bartz).

iteration procedure[10] (with quadratic convergence). This iteration scheme begins with an initial value of $\dot{q}_c$ and continues until $|F|$ is less than, say, 10^{-4}.

It is now simple to estimate the combustion gas temperature and heat-transfer coefficient in conjunction with the temperature history. The equation for converting the calculated heat flux to the heat-transfer coefficient is

$$h=\dot{q}_c/(T_g-T_0) \tag{6}$$

In the foregoing equation, T_g is an unknown quantity. For estimating this quantity, the finite-difference approximation at the boundary (surface) can be rewritten as

$$\left[-2-\frac{\Delta x^2}{\alpha\Delta t}-\frac{2\Delta x\dot{q}_c}{(T_g-T_0)k}\right]T_0^n+2T_1^n=-\frac{\Delta x^2}{\alpha\Delta t}T_0^{n-1}-\frac{2\Delta x\dot{q}_c}{k} \tag{7}$$

The combustion gas temperature can be obtained by using the above-mentioned minimization technique. The iteration scheme starts with an initial value of T_g and continues until $|F|$ is less than, say, 10^{-4}. The method estimates the component of $\dot{q}_c$ and T_g one at a time and thus may be considered an on-line method.[6]

Numerical difficulties arise in determining the surface heat rate and heat-transfer coefficient from data based on interior thermocouples. These difficulties are attributed to the time lag imposed on the system resulting from the surface and the thermocouple location.

Example

The iteration procedure discussed in the previous section has been utilized for estimating wall heat flux, surface temperature, convective heat-transfer coefficient, and combustion gas temperature for a typical divergent rocket nozzle made of mild steel in conjunction with experimentally measured outer surface temperature data in a static test. The nozzle conditions and material properties taken are $L=0.0211$ m, initial temperature = 300 K, thermal conductivity (average) = 35 W/m·K, $\alpha=8.1291\times10^{-4}$ m^2/s, and burning time = 16 s. Combustion gas temperature T_{gc} is obtained from thermodynamic calculations. This inverse program, in turn, utilized these transient data to determine unknown surface conditions. Twenty space intervals and a time increment of 1 s are used for computational purposes. Initial time of 6 s is taken for starting the solution. It is seen from Table 1 that the estimated value of the convective heat transfer is somewhat lower (except at 6 s) than the calculated results of Bartz.[11] The percentage error, $[(T_g-T_{gc})/T_{gc}]\times100$, between the estimated value of T_g and T_{gc} is found to be 13.76 at $t=6$ s; at $t>6$ s it varies in the range of 5.80-0.41%. These disagreements are attributed to higher initial time step for starting the solution. The errors may also arise due to the method used for approximating the thermal model, the magnitude of $\dot{q}_c$, and the magnitude of the thermocouple temperatures, as well as the location of the thermocouple in relation to the surface. However, it is also important to mention here that the Bartz equation gives a conservative estimation for heat-transfer coefficient.[10,12]

Conclusions

The Newton-Raphson iteration procedure proves quite useful in estimating the values of wall heat flux, surface temperature, convective heat-transfer coefficient, and combustion gas temperature from the measured thermocouple temperature on the outer surface of the nozzle. The advantage in using this numerical approach lies in its capability of handling the irregular behavior of the surface heat flux.

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